Experimental Tetrahedral Recording: Part Three

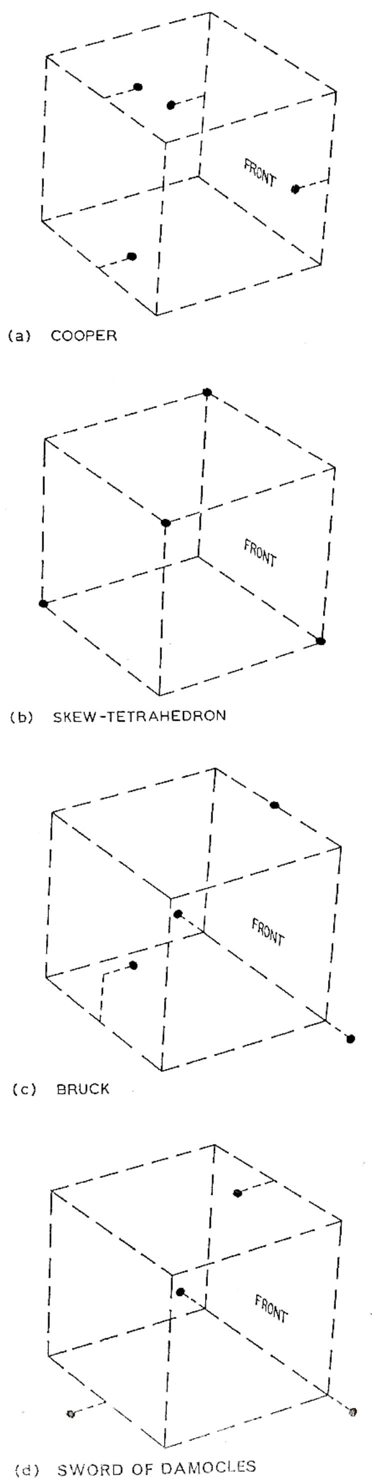
By Michael Gerzon

IN the previous two parts of this series, the method of making tetrahedral recordings was described. This last part describes the possible uses of such recordings in a wide variety of playback experiments.

If the precautions outlined in Part Two have been followed, the recording should consist of four coincident cardioid (or hypercardioid) signals pointing to the four corners of the cube shown in **fig. 1b**: L_R to rear left downward, L_F to front left upwards, R_F to front right downwards, and R_R to rear right upwards. By matrixing these four signals, it is possible to obtain *any* conventional microphone characteristic output pointing in *any* direction.

A possible adjustable matrixing circuit, in this case with four inputs and two outputs, is illustrated in **fig.2**. As many extra outputs as desired may be added, as long as the input impedance does not become too low for the input signals. The gain of each of the inputs on a given output is varied between +1 unit and -1 unit by means of the potentiometers VR and the phase controls S. While transformers are shown in the schematic of **fig. 2**, transistorised

FIG. 1 REGULAR TETRAHEDRAL LOUDSPEAKER LAYOUTS SHOWN EMBEDDED IN A CUBE WHOSE SIDE LENGTH EQUALS THE HEIGHT OF THE LAYOUT



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phase-splitter circuitry is cheaper and potentially better. A considerable loss of voltage is caused by the isolating resistors R (about 12 dB with four outputs) and it is recommended that an amplifying stage be incorporated at each output unless low capacitance signal leads are used. It is not recommended that the gain controls be placed between the + and - lines of the input transformers, although this would obviate the need for a phase switch as it would either decrease the input impedance or increase the non-linearity and interaction of the controls, or both.

By this or suitable alternative means, it is possible to derive any combination of the four input signals, and the coefficients of each input signal can be set directly on the controls VR. This allows the matrixing circuit to be adjusted instantly for any possible experimental requirement, as long as the coefficients that should occur in the matrix are known. For this reason, most of the rest of this article is devoted to giving the matrixings required to derive various different types of signals from the standard tetrahedral recording.

For reasons of space and convenience of presentation, we shall use standard matrix notation for this. For those not familiar with matrix notation, a given signal in the left column of these tables is equal to that combination of the signals in the right column with the coefficients in the given signal's row of numbers. (In addition, negative numbers have been indicated here by underlining instead of by the more usual minus sign.) Thus, for example, in **Table 1c**, the signal L_B is given by:
$L_B = 0.663\ L_R + 0.544\ L_F + 0.245\ R_F - 0.452\ R_R$ and in **Table 3** the signal A_6 is given by:
$A_6 = 0.483\ L_R - 0.483\ L_F + 0.629\ R_F + 0.371\ R_R$.

Table 1 gives the matrixings required to convert a skew tetrahedral recording with signals L_R, L_F, R_F, R_R as in **fig. 1b** into a recording intended for reproduction via one of the other tetrahedral layouts. The matrixings given synthesise microphone outputs pointing along the relevant tetrahedral axes with the same microphone directional characteristic as used for the original recording.

The outputs L_C (front left), R_C (front right), A_C (rear above) and B_C (rear below) for the Cooper tetrahedral speaker layout of **fig. 1a** may be derived as in **Table 1a**. The outputs T_B (front top), L_B (front left), R_B

(front right) and B_B (back) for the Bruck speaker layout of **fig. 1c** are derived as in **Table 1c**. The outputs T_D (top), L_D (front left), R_D (front right) and B_D (back) for the 'sword of Damocles' layout of **fig. 1d** may be derived as in **Table 1d**. The outputs L_R*, L_F*, R_F*, R_R* for the skew tetrahedral system with speakers in the *lower* front left and right rear positions, and in the *upper* left rear and right front positions, may be derived as in **Table 1b**.

Those with a knowledge of matrix algebra should note that the matrices of **Table 1** are orthogonal, so that to convert the other way (e.g. from a Cooper to a skew tetrahedral recording), the inverse matrix may be obtained simply by writing down the transpose, i.e. interchanging rows and columns. Similarly, to obtain the matrixing from one of these systems to another (e.g. Cooper to Bruck), simply compute the matrix $B\ C^T$, where C is the matrix given in **Table 1** for the system (e.g. Cooper) which is being converted, and B is the matrix in table for the system (e.g. Bruck) to which it is being changed. To facilitate such computations, all coefficients have been given to three decimal places.

It may prove necessary to rotate the stereo image because of inaccurate microphone placement, or to bring sounds to the front of the listener. **Table 2a** gives the matrixing for the skew tetrahedral system that rotates the image horizontally around the listener by an angle θ clockwise. If θ is made negative, then the image is rotated anticlockwise. Note that a clockwise rotation of the image is also produced by an *anticlockwise* rotation of the original microphones. **Table 2b** gives the matrixing for the skew tetrahedral system that rotates the stereo image by an angle θ upwards at the front about the axis running through the listener's ears. This should prove useful with recordings made with high-up microphones. **Table 2c** gives the matrixing for the skew tetrahedral system that rotates the stereo image by an angle θ clockwise about the front-back axis. This should prove useful for correcting tilted microphones.

Again, by applying matrix algebra methods, it is possible to compute the method of rotating the sound for recordings made for the Cooper, Bruck or Damocles layouts. For example, if H is the matrix corresponding to the desired rotation in **Table 2**, and if B is the conversion matrix corresponding to the Bruck system in **Table 1c**, then the matrix producing the same rotation for Bruck-system recording is BHB^T.

Besides tetrahedral methods of playback, the four-channel 'tetrahedral' recording can also be used for playback over more complex loudspeaker layouts. For example, the sound can be played over a cube of eight speakers, placed at the cube corners of **fig. 1b**, by feeding to them the eight signals L_R, L_R*, L_F, L_F*, R_F, R_F*, R_R and R_R* (see **Table 1b**) in the obvious manner (see also ref. 1).

Another method of playback is over six loudspeakers arranged to form a regular octahedron around the listener. While there are many possible octahedral speaker layouts, the best stereo image will be obtained only if all six speakers lie at the same angle off the axis through the two ears of the listener. This suggests that the octahedral loudspeaker layout of **fig. 3** should be used (or else its mirror-image). The signal fed to each loudspeaker will be the signal that would have been picked up by a cardioid microphone pointing in its direction. If the six loudspeakers are labelled A_1 to A_6 as illustrated, then their signals may be derived from the usual skew tetrahedral recording by the matrixing given in **Table 3**.

With the possibility of such loudspeaker layouts, it will be seen that the name 'tetrahedral stereo' is rather a misnomer, for the only way that a tetrahedron enters into such recordings is in the tetrahedral axes that happen to be chosen for the signals fed to the four tracks of the tape. There are many possible ways of storing the four parameters of information that determine the three-dimensional direction effect around the microphones, of which an alternative method would be to record the outputs of an omnidirectional microphone and that of three mutually perpendicular figure-of-eights. Because tetrahedra are involved only in describing the *way* the information is stored on the tape, and have nothing to do with the *content* of this information, the author has proposed that systems of recording the full directional effect around the microphones, including height, should be called *periphonic systems* (*peri-*, around, Greek).

The reader will see that, in principle, any regular or almost regular loudspeaker layout can be used for periphonic reproduction—tetrahedron, octahedron, cuboctahedron, dodecahedron, icosahedron, or even 62 speakers placed on the 31 axes of icosahedral symmetry. I do not propose to give the matrixing for the latter here. Perhaps the most important area of research is to determine methods of reproducing the correct directional effect over non-regular loudspeaker layouts, as only such irregular layouts stand a chance of fitting conveniently into a wide variety of domestic furnishing schemes.

All the matrixings of **Tables 1, 2** and **3** have the effect of converting four signals picked up with four identical cardioid or hypercardioid microphones pointing in four different directions into new signals which are effectively picked up by the same cardioid or hypercardioid characteristic pointing in four (or six) new directions. It may be desired to change the shape of the microphone pick-up characteristic to reduce overlap (see Part One). In this case, one uses the matrixings given in, or computed from, **Tables 1, 2** and **3**, except that a small constant is added to or subtracted from every coefficient in the matrixing. For example, if one wishes to convert from cardioid to 135°-null hypercardioid, while performing one of the operations described in tables 1-3, one uses a matrixing in which every coefficient is 0.073 smaller that it would be if a cardioid characteristic were retained. Similarly, if the original recording is made with four 135°–null hypercardioids (possibly due to pre-record matrixing) then 0.104 must be added to each matrix coefficient to restore cardioid outputs. To convert a cardioid recording to, respectively, 150°, 135° and 125°–null hypercardioids, one must subtract 0.033, 0.073 and 0.107 from every matrix coefficient in **Tables 1, 2** or **3**.

It is possible to rematrix tetrahedral recordings to throw away the height information (see ref. 1). This may be useful if it is desired to determine the subjective importance of the height information, and the relevant matrixing is given in **Table 4a**.

There is the related problem of playing 'conventional' four-channel recordings via a skew-tetrahedral speaker layout so that all sounds come from a horizontal direction. This also entails throwing out the spurious height information, but the matrixing of **Table 4a** cannot be used in this case, as it would cause undesirable out-of-phase images because conventional four-channel recordings are not properly 'conditioned' for the requirements of tetrahedral playback. The matrixing of **Table 4b** is a compromise that may allow

conventional four channel recordings to be reproduced tetrahedrally.
Table 4c gives a matrixing that may allow a conventional four channel recording to be reproduced over the octahedral layout of **fig. 3**.

We may play two channel stereo recordings via the skew tetrahedral or octahedral layouts by using the matrixings of **Table 4b** or **4c**, treating the stereo signal as if it were the front two channels of a four channel recording. Hafler-style surround-sound reproduction of two channel recordings is also possible. **Tables 4d** and **4e** give matrixings that may produce approximately horizontal surround-sound reproduction of two channels via the skew-tetrahedral and octahedral layouts.

Besides experiments with surround-sound and periphony, the other main use of tetrahedral recordings is in the study of ordinary two channel stereo microphone techniques. A tetrahedral recording made with coincident microphones contains within its four tracks sufficient information for any conventional coincident microphone recording to be reconstructed by matrixing. Thus, for the first time, it is possible to perform repeatable objective comparisons between the different microphone techniques, something which has not been done up to now as far as I am aware.

Table 5 gives the matrixings required to derive the left (L) and right (R) signals of various two channel recording techniques from a skew tetrahedral recording. Thus such a recording provides all the advantages of variable characteristic microphones, except that adjustments can be made after the recording, and a greater variety of adjustments are possible.

If a tetrahedral recording has been made with microphones that are spaced apart significantly, then much of the above matrixing will no longer work, and one is restricted to reproduction via one particular loudspeaker layout which may well prove to be non-optimum. Slightly modified matrixings could well give adequate results if the microphone spacing is moderate. A disadvantage of highly coincident microphones is that they tend to interfere with one another acoustically at high frequencies, but this is considered to be a relatively small price to pay for experimental flexibility.

The above account has only indicated a few of the many possible experimental uses of tetrahedral recordings, but nevertheless indicates just how much information is contained in the four channels. In a precisely definable sense, tetrahedral recording makes much more efficient use of four channels than any other current proposal. So great is the system's flexibility that a full appreciation of its uses and possibilities requires a more profound analysis than is possible in these pages. This flexibility is equally great whether coincident or multimike recording techniques are used (see ref. 1) A great deal of experimental work remains to be done before the system is ready for domestic use, as is apparent from the very large number of possible playback methods.

The intention of this series of three articles has been to set out the requirements and possibilities involved in tetrahedral recording, so that others should be encouraged to experiment with this technique. Like any new technology, the new recording system requires some unlearning of old tricks and the learning of new ones. With the extreme newness of even 'conventional' four channel stereo, it is hardly surprising that many of the old methods that worked with two channel stereo are still being applied erroneously to four channel systems. It is hoped that a study of ref. 1 and of this series of articles will have given some understanding of the special requirements of periphony. Finally, one must acknowledge the value of the pioneering tetrahedral recordings of Granville Cooper (ref. 2), whose hard-won experience has proved so useful in formulating the problems in doing experimental recordings.

References

1. Michael Gerzon, Principles of Quadraphonic Recording, part Two, STUDIO SOUND, September 1970
2. Granville Cooper, Tetrahedral Ambiophony, STUDIO SOUND, June 1970

[Tables 1-4 and Figures 2 and 3 are on the following pages]

TABLE 1 Matrixings converting standard skew tetrahedral signals of fig. 1b into other tetrahedral systems.

1a: Conversion to Cooper system.

$$\begin{bmatrix} L_C \\ R_C \\ A_C \\ B_C \end{bmatrix} = \begin{bmatrix} \cdot354 & \cdot854 & \cdot146 & \underline{\cdot354} \\ \underline{\cdot354} & \cdot146 & \cdot854 & \cdot354 \\ \cdot146 & \cdot354 & \underline{\cdot354} & \cdot854 \\ \cdot854 & \underline{\cdot354} & \cdot354 & \cdot146 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}$$

1b: Conversion to mirror-image skew tetrahedron.

$$\begin{bmatrix} L_R^* \\ L_F^* \\ R_F^* \\ R_R^* \end{bmatrix} = \begin{bmatrix} \cdot500 & \cdot500 & \underline{\cdot500} & \cdot500 \\ \cdot500 & \cdot500 & \cdot500 & \underline{\cdot500} \\ \underline{\cdot500} & \cdot500 & \cdot500 & \cdot500 \\ \cdot500 & \underline{\cdot500} & \cdot500 & \cdot500 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}$$

1c: Conversion to Bruck system.

$$\begin{bmatrix} T_B \\ L_B \\ R_B \\ B_B \end{bmatrix} = \begin{bmatrix} \underline{\cdot303} & \cdot803 & \underline{\cdot014} & \cdot514 \\ \cdot663 & \cdot544 & \cdot245 & \underline{\cdot452} \\ \underline{\cdot044} & \underline{\cdot163} & \cdot952 & \cdot255 \\ \cdot683 & \underline{\cdot183} & \underline{\cdot183} & \cdot683 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}$$

1d: Conversion to 'sword of Damocles' system.

$$\begin{bmatrix} T_D \\ L_D \\ R_D \\ B_D \end{bmatrix} = \begin{bmatrix} \underline{\cdot183} & \cdot683 & \underline{\cdot183} & \cdot683 \\ \cdot544 & \cdot663 & \cdot254 & \underline{\cdot452} \\ \underline{\cdot163} & \underline{\cdot044} & \cdot952 & \cdot255 \\ \cdot803 & \underline{\cdot303} & \underline{\cdot014} & \cdot514 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}$$

TABLE 2 Matrixings rotating skew tetrahedral signals

2a: Horizontal clockwise rotation by θ.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{out}} = \begin{bmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) & \frac{1}{2}\sin\theta \\ \frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}\sin\theta \\ -\frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) & \frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

2b: Rotation about ears' axis by θ upwards at front.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{out}} = \begin{bmatrix} \frac{1}{2}(1+\cos\theta) & \frac{1}{2}(1-\cos\theta) & -\frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{2}(1+\cos\theta) & \frac{1}{2}\sin\theta & -\frac{1}{2}\sin\theta \\ \frac{1}{2}\sin\theta & -\frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) & \frac{1}{2}(1-\cos\theta) \\ -\frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) & \frac{1}{2}(1+\cos\theta) \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

2c: Rotation clockwise about front-back axis by θ.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{out}} = \begin{bmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}\sin\theta & \frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) \\ \frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) & \frac{1}{2}(1-\cos\theta) & -\frac{1}{2}\sin\theta \\ -\frac{1}{2}\sin\theta & \frac{1}{2}(1-\cos\theta) & \frac{1}{2}(1+\cos\theta) & \frac{1}{2}\sin\theta \\ \frac{1}{2}(1-\cos\theta) & \frac{1}{2}\sin\theta & -\frac{1}{2}\sin\theta & \frac{1}{2}(1+\cos\theta) \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

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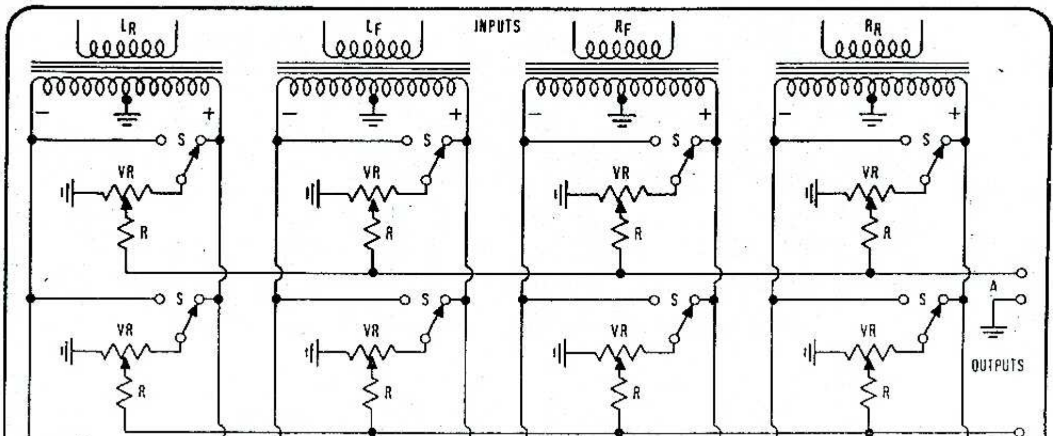

FIG. 2
FULLY VARIABLE MATRIXING CIRCUIT R MUST BE GREATER THAN 10 VR

FIG. 3 OCTAHEDRAL LOUDSPEAKER LAYOUT
ALL DISTANCES SHOWN AS A MULTIPLE OF THE STRAIGHT-LINE DISTANCE BETWEEN ADJACENT SPEAKERS.
SPEAKERS A_2 AND A_5 LIE STRAIGHT ABOVE A_3 AND A_6 RESPECTIVELY

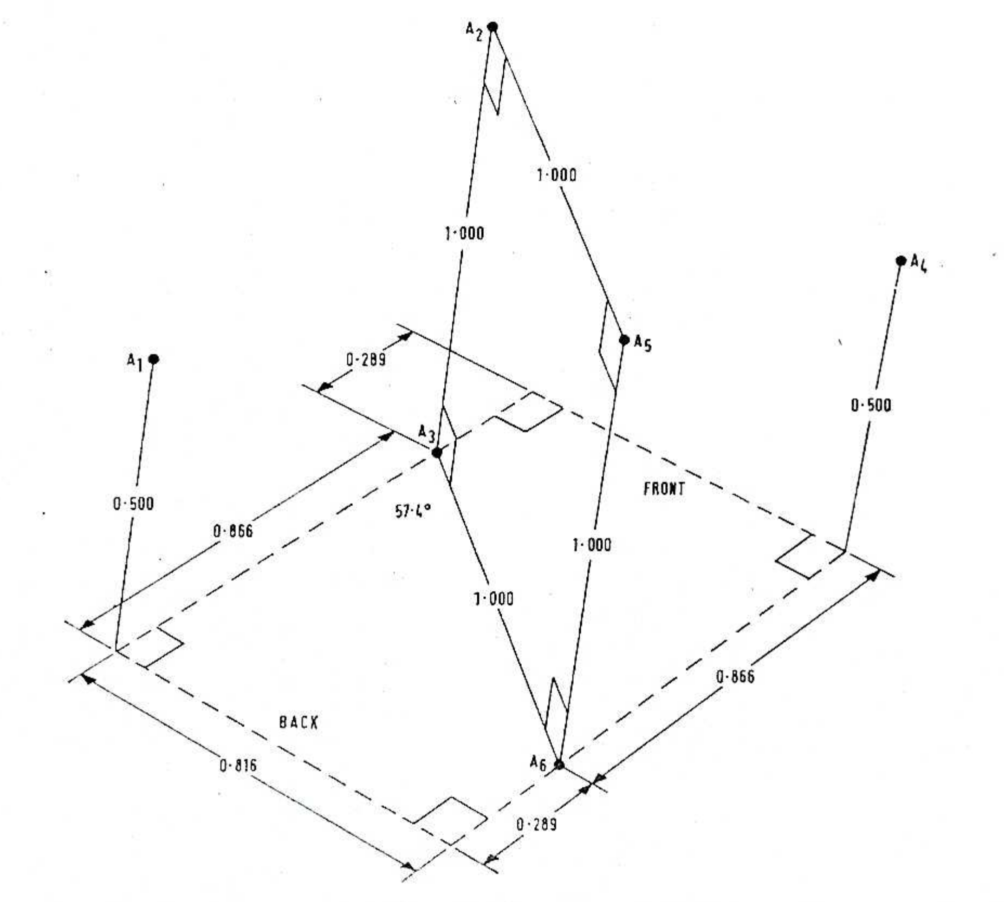

TABLE 3 Conversion from the skew tetrahedral system of fig. 1b to the octahedral system of fig. 3.

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} \cdot854 & \cdot146 & \underline{\cdot354} & \cdot354 \\ \cdot017 & \cdot983 & \underline{\cdot129} & \cdot129 \\ \cdot629 & \cdot371 & \cdot483 & \underline{\cdot483} \\ \underline{\cdot354} & \cdot354 & \cdot854 & \cdot146 \\ \underline{\cdot129} & \cdot129 & \cdot017 & \cdot983 \\ \cdot483 & \underline{\cdot483} & \cdot629 & \cdot371 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}$$

TABLE 4 Matrixings for reproducing 'conventional' recordings via skew-tetrahedral and octahedral speaker layouts

4a: Suppression of height information in tetrahedral recordings.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{out}} = \begin{bmatrix} 1 & \cdot333 & \underline{\cdot333} & \cdot333 \\ \cdot333 & 1 & \cdot333 & \underline{\cdot333} \\ \underline{\cdot333} & \cdot333 & 1 & \cdot333 \\ \cdot333 & \underline{\cdot333} & \cdot333 & 1 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

4b: Matrixing for tetrahedral playback of 'conventional' 4-channel recording.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{out}} = \begin{bmatrix} 1 & \cdot414 & \underline{\cdot172} & \cdot414 \\ \cdot414 & 1 & \cdot414 & \underline{\cdot172} \\ \underline{\cdot172} & \cdot414 & 1 & \cdot414 \\ \cdot414 & \underline{\cdot172} & \cdot414 & 1 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

4c: Matrixing for octahedral playback of 'conventional' 4-channel recording.

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdot172 \\ \cdot121 & \cdot707 & 0 & 0 \\ \cdot121 & \cdot707 & 0 & 0 \\ 0 & \cdot172 & 1 & 0 \\ 0 & 0 & \cdot121 & \cdot707 \\ 0 & 0 & \cdot121 & \cdot707 \end{bmatrix} \begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix}_{\text{in}}$$

4d: Surround-sound reproduction of 2 channels via a skew tetrahedron.

$$\begin{bmatrix} L_R \\ L_F \\ R_F \\ R_R \end{bmatrix} = \begin{bmatrix} 1 & \underline{\cdot414} \\ 1 & \cdot414 \\ \cdot414 & 1 \\ \underline{\cdot414} & 1 \end{bmatrix} \begin{bmatrix} L \\ R \end{bmatrix}$$

4e: Surround-sound reproduction of 2 channels via an octahedron.

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} = \begin{bmatrix} \cdot888 & \underline{\cdot460} \\ \cdot674 & \cdot214 \\ \cdot674 & \cdot214 \\ \cdot460 & \cdot888 \\ \underline{\cdot214} & \cdot674 \\ \underline{\cdot214} & \cdot674 \end{bmatrix} \begin{bmatrix} L \\ R \end{bmatrix}$$