

The Principles of Quadraphonic Recording

Part Two: The Vertical Element

By Michael Gerzon

CURRENT quadraphonic systems are designed to reproduce sound from all horizontal directions around the listener, but still fail to reproduce height information. In Part One of this article, by means of considering the types of sound pick-up associated with the use of a coincident microphone technique, it was shown that only three channels were necessary for 'horizontal quadraphony'. In the following, these arguments will be extended to the reproduction of sound from all spatial directions about the listener, both horizontally and vertically. The author has christened reproduction techniques which reproduce all spatial directions 'periphonic' from the Greek prefix *peri-* meaning about, or around.

While Granville Cooper has recently described³ a system of periphony called 'tetrahedral ambiophony', this is only one of many possible periphonic techniques. It is the purpose of this article to establish that four channels should usually be adequate to convey periphonic sound. It will further be shown that it is possible to convey a periphonic recording via four channels such that, when it is reproduced through four loudspeakers placed in a horizontal square around the listener (as in current American "horizontal quadraphony" proposals), a good conventional quadraphonic sound reproduction will be obtained. Thus the method of conveying periphony described in the following has the advantage of complete compatibility with conventional quadraphonic reproduction. A consequence of this is that the listener has a wide choice as to how complex his reproduction system is, and he may choose to reproduce the four channels over anything between three and eight speakers, according to his pocket and preferences.

First question

The first question to be resolved is why reproduce height information at all? The case against periphony has been wittily stated by Alec Nisbett, and it is worth quoting him:⁴

"I am not being totally facetious when I suggest that if God had meant us to take an interest in the vertical separation of sounds, we would have an ear on the top of our heads. Lacking such a rainwater collector, I don't see much need to feed directional information in this sense, even though it is present in the concert hall: the horizontal component is enough. Anyway, I am going to cut short this argument by saying that if you don't like the horizontal box format it's just too bad, because that's how it's going to be – there's no turning back, unless you want another big battle over standards, which would be exhausting, expensive and, I suspect, unwinnable. So everybody please agree with me."

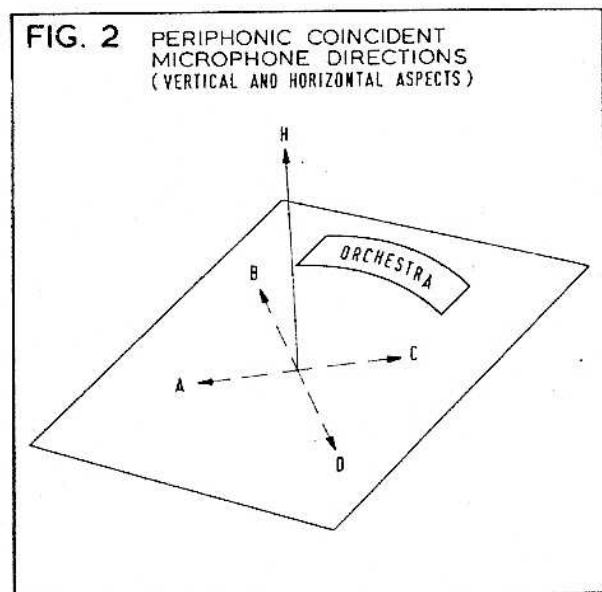
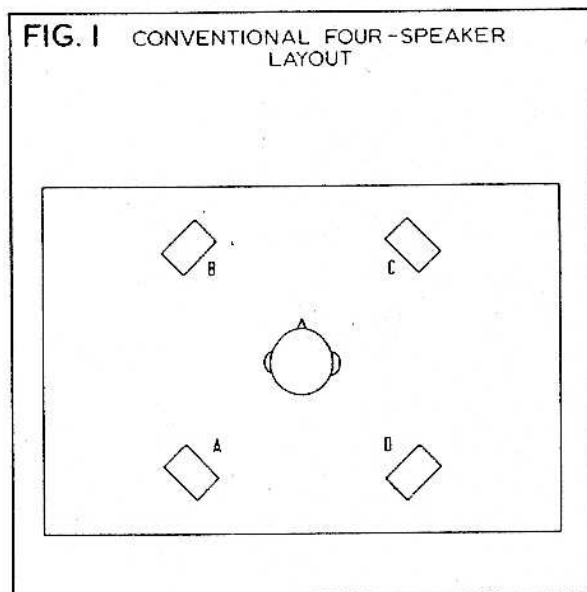
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The fallacy with the argument is the assumption that human hearing is insensitive to height information. It is well known that it is possible to perceive the elevation of a sound quite accurately by means of small unconscious head movements.⁵

In the author's experience, height information can be of great musical importance. In orchestral and choral music, a strong impression of depth is often gained due to the fact that the orchestra frequently subtends a vertical angle of a few degrees at the listener's ears; this height information is then clearly audible with one's eyes shut. Of even greater musical importance is the existence of religious and secular music in which a large organ accompanies a choir or orchestra. In this case, composers have often (perhaps not consciously?) used the fact that the organ will be placed high up above the other performers to obtain a remote, all pervading, or ethereal effect from the organ. This effect is totally destroyed by restricting the sound to the horizontal plane. It should also be mentioned that, even using the realistic coincident microphone technique, reverberation sounds curiously 'cramped' via horizontal quadraphony, due to the lack of height dimension.

Alec Nisbett's other objection, that it is undesirable and impractical to introduce more than one system of quadraphony, ceases to hold if the periphonic recording is capable of being reproduced via a conventional quadraphonic set-up. This compatibility requirement can be completely fulfilled, as will be shown in the following.

Consider a conventional four-channel quadraphonic recording with signals A, B, C, and D corresponding to the four loudspeakers placed in a horizontal square about the listener, as in **fig.1**. It was shown in Part One that a good quadraphonic sound could be obtained even if $\frac{1}{2}A - \frac{1}{2}B + \frac{1}{2}C - \frac{1}{2}D$ was equal to zero, and methods were described to convert arbitrary four-channel recordings into a form where this was



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true. Thus, in the rest of this article, we consider quadraphonic signals A, B, C, and D such that

$$\frac{1}{2}A - \frac{1}{2}B + \frac{1}{2}C - \frac{1}{2}D = 0 \quad (1)$$

By imposing the condition (1) on our quadraphonic signals, we have produced signals that can be conveyed via only three channels, as explained in Part One.

Thus, if a four-channel recording medium is used, there is room to convey height information. Let H be a "height" audio signal, whose precise nature will be considered later. Then one can make a four-channel recording conveying the four signals

$$\begin{aligned} A^- &= A - \frac{1}{2}H, & B^+ &= B + \frac{1}{2}H, \\ C^- &= C - \frac{1}{2}H, & D^+ &= D + \frac{1}{2}H. \end{aligned} \quad (2)$$

The four signals A^- , B^+ , C^- , and D^+ may be reproduced via the horizontal four-speaker set-up of **fig.1** without any alteration of the directional effect that would have been reproduced if A, B, C and D had been fed to the four speakers instead. The reason for this is that the 'focus' signal F' for the four signals A^- , B^+ , C^- , and D^+ is given by

$$F' = \frac{1}{2}A^- - \frac{1}{2}B^+ + \frac{1}{2}C^- - \frac{1}{2}D^+ = -H. \quad (3)$$

As was shown in Part One, altering the focus signal in a four-channel quadraphonic recording does not alter the reproduced directional effect, but only affects the degree of crosstalk of a sound on to the other channels.

Thus, if we start off with a conventional quadraphonic recording whose signals A, B, C, D obey condition (1) and if we smuggle in a "height" signal H as in formulae (2), then we have four signals which reproduce well via a conventional four-speaker set-up, but which contain height information as well as horizontal information.

Of course, this has not yet shown either how to record the height information, nor how to reproduce it. As in Part One, examining coincident microphone recording techniques is very revealing. Only microphones with horizontally-pointed axes were then considered but we must now consider coincident microphones with axes pointing in any direction. Standard mathematical theory reveals that there are only four linearly independent microphone directional characteristics. Put another way, given five coincident microphones, it is always possible to derive the audio output of at least one of the microphones by matrixing the outputs of the other four microphones together in suitable proportions.

This means that no matter how many loudspeakers are used to reproduce the sound, only four microphones are needed to pick up all the periphonic audio information that can be obtained from coincident microphones. (Of course, this no longer holds if the microphones are not precisely coincident. Neither will it hold if new types of microphone directional characteristics are developed.) The sound fed to each of the reproducing loudspeakers may be obtained by a suitable matrixing of the signals from these four microphones.

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The nature of the four signals A^- , B^+ , C^- , and D^+ will now be investigated for coincident microphone periphonic recordings. This investigation yields useful information on how to reduce spaced microphone periphonic recordings down to four channels.

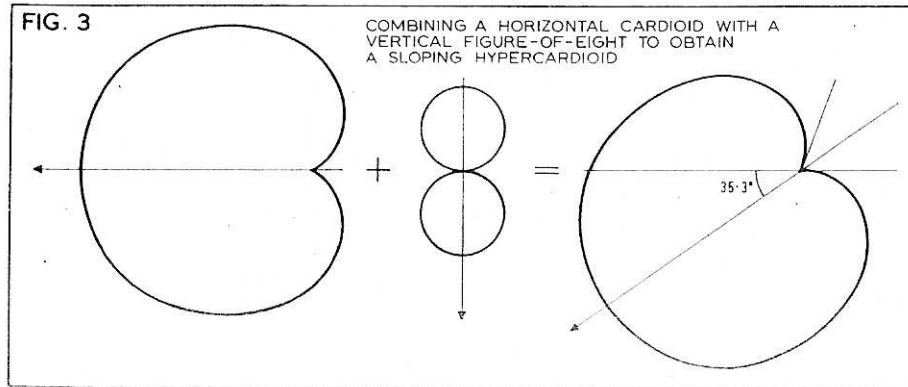
Clearly there are two ways of looking at the recording of the signals A^- , B^+ , C^- , and D^+ via coincident microphones. We can either consider the microphone directional characteristics required to pick up the signals A , B , C , D , and H , or we can consider the microphone directional characteristics required to pick up the signals A^- , B^+ , C^- , and D^+ . The first way of considering coincident microphone periphonic recording has the advantage that it lays special emphasis on the 'horizontal quadraphony' component A , B , C , D of the periphonic signal, while the second approach reveals the essentially three-dimensional geometric nature of periphonic recording.

As the signals A , B , C , and D correspond to horizontal quadraphonic sound they must be the signals obtained by four coincident identical cardioid or hypercardioid microphones whose axes point in a horizontal direction along four directions at right angles to each other as illustrated in **fig. 2**. Four such signals obey condition (1), as observed in Part One. Sounds originating from horizontal directions around the microphones clearly contain no height information, and so require that the signal H equals zero for such sounds. The only microphone directional characteristics which gives no output for sounds from all horizontal directions is a vertically oriented 'figure-of-eight' microphone. It is convenient to assume that the 'positive' lobe of the figure of eight points upwards, rather than downwards.

Thus, for coincident microphone recordings, the four signals conveying the periphonic sound are $A^- = A - \frac{1}{2}H$, $B^+ = B + \frac{1}{2}H$, $C^- = C - \frac{1}{2}H$, $D^+ = D + \frac{1}{2}H$, where A , B , C , and D are the conventional quadraphonic signals obtained by identical coincident horizontal cardioid or hypercardioid microphones, and where H is the output of an upward pointing figure-of-eight microphone. However, this has not completely specified the nature of the periphonic signal A^- , B^+ , C^- , and D^+ for coincident microphones, as we do not yet know the correct relative gains of the H signal and the A , B , C , and D signals. Put crudely, how loud should the height signal be compared to horizontal quadraphonic signal in formulae (2)?

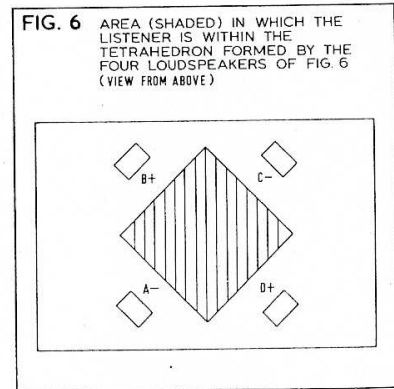
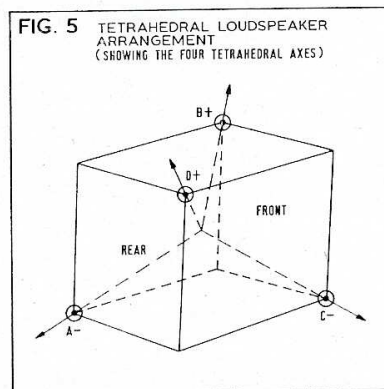
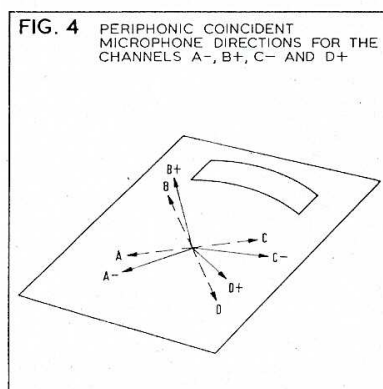
To answer this, we look at the microphone directional characteristic required to pick up the signals A^- , B^+ , C^- , and D^+ . Each of these signals is obtained by adding the audio output of a horizontal cardioid or hypercardioid microphone to that of a vertical figure-of-eight microphone. Thus one may consider that the signals A^- , B^+ , C^- , and D^+ are picked up by hypercardioid microphones pointing at an angle to the horizontal (see **fig.3**). Thus the signals B^+ and D^+ are the signals that would be picked up by hypercardioid microphones pointing at an angle *above* the horizontal B and D

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directions, and the signals A^- and C^- are the signals that would be obtained by hypercardioid microphones pointing at an angle *below* the A and C directions, as illustrated in **fig. 4**.

It is desirable that the four directions in which the A^- , B^+ , C^- , and D^+ microphones point should be disposed as symmetrically as possible, and the most symmetrical arrangement possible is obtained if the four microphones point along the axes of a regular tetrahedron. (A tetrahedron is said to be *regular* if all its sides are equal.) This requirement is fulfilled if the axes of these four microphones are inclined at an angle of 35.3° to the horizontal. (This occurs if the sensitivity of the vertical figure-of-eight picking up H is $\sqrt{2}$ times the sensitivity of the figure-of-eight component of the horizontal directional characteristics used to pick up A , B , C , or D , ignoring the omnidirectional component.)



Thus the four periphonic signals A^- , B^+ , C^- , and D^+ are required to be the signals picked up by four identical coincident hypercardioid microphones directed along tetrahedral axes pointing in a direction 35.3° above (in the case of B^+ and D^+) or below (in the case of A^- and C^-) the horizontal directions A , B , C , and D illustrated in **figs. 2** and **4**. The requirements of simplicity and compatibility with conventional quadraphony have thus led to a periphonic recording system in which four identical hypercardioid microphones point along four tetrahedral axes.

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The proposal for periphonic recording is very similar to Granville Cooper's,³ except that in our case the axes of the tetrahedron point in different directions. The axes of the four microphones picking up A⁻, B⁺, C⁻, and D⁺ point along the lines connecting the centre of a cube to four of its eight corners, as illustrated in **fig. 5**.

The simplest method of reproducing the original directional effect of the periphonic signals A⁻, B⁺, C⁻, and D⁺ is to feed them to four loudspeakers placed at four corners of a cube, as illustrated in **fig. 5**. A⁻ is fed to a floor-level rear left speaker, B⁺ is fed to a ceiling-level front left speaker, C⁻ is fed to a floor-level front right speaker, and D⁺ is fed to a ceiling level rear right speaker. For a given room height this tetrahedral speaker layout (and its mirror image) encloses a larger volume than any other possible arrangement using four loudspeakers placed at the corners of a regular tetrahedron. For this reason, the listening area in which reasonable periphonic reproduction can be obtained is likely to be larger than with any other tetrahedral arrangement of loudspeakers, including that of Granville Cooper.³ For a listener whose ears are half way between the floor and ceiling, the portion of the room in which his head lies within the tetrahedron is indicated by the shaded area of **fig. 6**. Within this area, a reasonable periphonic effect should be obtained, although this has not been tried experimentally.

Reproduced sounds

Reproduced sounds will appear to come from a horizontal direction via the loudspeaker layout of **fig. 5** only if the signals A⁻, B⁺, C⁻, and D⁺ contain no height information. This occurs when the focus signal $\frac{1}{2}A^- - \frac{1}{2}B^+ + \frac{1}{2}C^- - \frac{1}{2}D^+$ is equal to zero. Thus a coincident-microphone horizontal quadraphonic recording will reproduce well over the **fig. 5** tetrahedral loudspeaker layout. However, conventional two-channel stereo or spaced-mike horizontal quadraphonic recordings will reproduce properly via this speaker layout only if their focus information is suppressed. As explained last month, several different matrixings are capable of suppressing the focus. Tables 8 and 9 give a typical matrixing that allows conventional stereo and horizontal quadraphony to be reproduced via tetrahedral loudspeakers. In the case of ordinary stereo, it will be seen that the tilt of the sound from the front speakers is compensated for by the opposing tilt of the rear speakers.

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TABLE 8 Playing a stereo signal L and R through the tetrahedral speaker layout of fig. 6

$$\begin{aligned} A^- &= 0.354 L - 0.146 R \\ B^+ &= 0.854 L + 0.354 R \\ C^- &= 0.354 L + 0.854 R \\ D^+ &= -0.146 L + 0.354 R \end{aligned}$$

TABLE 9 Playing a horizontal quadraphonic signal A^1 , B^1 , C^1 , and D^1 through the tetrahedral speaker layout of fig. 6.

$$\begin{aligned} A^- &= 0.854 A^1 + 0.354 B^1 - 0.146 C^1 + 0.354 D^1 \\ B^+ &= 0.354 A^1 + 0.854 B^1 + 0.354 C^1 - 0.146 D^1 \\ C^- &= -0.146 A^1 + 0.354 B^1 + 0.854 C^1 + 0.354 D^1 \\ D^+ &= 0.354 A^1 - 0.146 B^1 + 0.354 C^1 + 0.854 D^1 \end{aligned}$$

TABLE 10 Corresponding horizontal and tetrahedral microphone directional pick-up characteristics (see text).

HORIZONTAL MICROPHONES		TETRAHEDRAL MICROPHONES	
Angle of null off axis	Front-to-back ratio	Angle of null off axis	Front-to-back ratio
Not hypercardioid	19.91 dB	180° *	∞ dB *
180° *	∞ dB *	144.7°	19.91 dB
150°	22.88 dB	135°	15.31 dB
135°	15.31 dB	125.3°	11.44 dB

*N.B. – cardioid directional characteristic

The identical cardioid or hypercardioid directional characteristics used to pick up the horizontal signals A, B, C, and D are not the same as the identical hypercardioid directional characteristics used to pick up the signals A^- , B^+ , C^- , and D^+ , due to the fact that the latter contain a proportion of the vertical figure-of-eight H signal. A hypercardioid directional characteristic may be specified either by its front-to-back ratio or by the angle from its axis at which its null response lies. Table 10 indicates the hypercardioid characteristics ('tetrahedral microphones') used to obtain A^- , B^+ , C^- , and D^+ corresponding to each of a range of possible hypercardioid characteristics ('horizontal microphones') used to pick up the signals A, B, C, and D. It will be seen

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that the microphone characteristics used to pick up A^- , B^+ , C^- , and D^+ are more hypercardioid, and less cardioid, than the corresponding microphone characteristics used to pick up A , B , C , and D .

Ideally, the microphone directional characteristics used to pick up A^- , B^+ , C^- , and D^+ should have a good front-to-back ratio so as to prevent sounds being reproduced loudly from loudspeakers in the direction opposite to that from which the sound should appear to come. This requirement would imply that the microphone characteristics used to pick up the signals A^- , B^+ , C^- , and D^+ should be cardioids, as in Granville Cooper's experimental recordings. However, it will be seen from Table 10 that the corresponding horizontal microphone pick-up is something between cardioid and omnidirectional, which means that horizontal sounds would be picked up with rather a lot of inter-speaker cross-talk. Also, for sounds originating on one of the tetrahedral axes, each of the three speakers corresponding to the other three axes in **fig. 5** would reproduce such sounds only 9.54 dB more quietly than the main speaker, and a quarter of the total audio energy would be reproduced from directions on the opposite side of the listener to the desired direction.

To reduce these effects, some degree of compromise between inter-speaker cross-talk and good front-to-back ratio has to be adopted, and it may be that a 135° -null hypercardioid characteristic for the A^- , B^+ , C^- , and D^+ microphones (or, equivalently, a 150° -null hypercardioid for the horizontal pick-up) will be a good compromise. In this case, the cross-talk of a sound appearing to come from one of the tetrahedral loudspeakers on to each of the other three loudspeakers is -13.19 dB. With 135° -null hypercardioids, only 0.026 of the energy of a sound being reproduced from a direction *opposite* to that of one of the tetrahedral axes will be reproduced from the speaker on that tetrahedral axis.

An alternative loudspeaker layout for periphonic reproduction might include eight loudspeakers arranged in a cube around the listener, as illustrated in **fig. 7**. When reproducing coincident microphone recordings, each of the eight speakers should be fed with the output that would be given by a hypercardioid microphone pointing in that speaker's direction. Labelling the speakers A^- , A^+ , B^- , B^+ , C^- , C^+ , D^- , and D^+ in the obvious way, the signals fed to the eight speakers will be:

$$A^-, A^+ = \frac{1}{2}A^- + \frac{1}{2}B^+ - \frac{1}{2}C^- + \frac{1}{2}D^+$$

$$B^-, B^+ = \frac{1}{2}A^- + \frac{1}{2}B^+ + \frac{1}{2}C^- - \frac{1}{2}D^+$$

$$C^-, C^+ = -\frac{1}{2}A^- + \frac{1}{2}B^+ + \frac{1}{2}C^- + \frac{1}{2}D^+$$

$$D^-, D^+ = \frac{1}{2}A^- - \frac{1}{2}B^+ + \frac{1}{2}C^- + \frac{1}{2}D^+$$

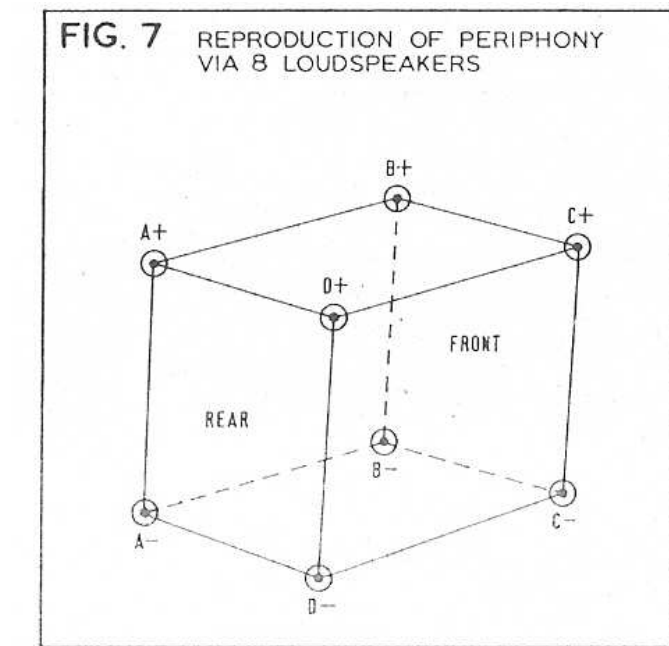
and D^+ respectively. (This may be seen by using formulae (2) and (3) along with the obvious fact that

$$A^+ = A + \frac{1}{2}H$$

$$B^- = B - \frac{1}{2}H$$

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$C^+ = C + \frac{1}{2}H$
and $D^- = D - \frac{1}{2}H$.)



One advantage of eight-speaker periphonic reproduction (apart from the profits for speaker and amplifier manufacturers!) is the fact that conventional horizontal quadraphony can be reproduced without loss. The four-speaker tetrahedral layout of **fig. 5** can only reproduce horizontal quadraphony by suppressing its 'focus' information, as in the matrixing of Table 9. Another advantage of eight speakers is that the angle subtended between adjacent speakers at the listeners' ears is only 70.5° , as compared with 109.5° for the tetrahedral layout, and this should help to make stereo images more precise.

Clearly, much ingenuity could be expended devising various advantageous loudspeaker layouts using five, six or seven loudspeakers. For this reason, it is not proposed to investigate further loudspeaker layouts here.

The above has only discussed coincident microphone recordings, and if results are to be good with various different loudspeaker arrangements, the microphones have to be pretty coincident, with spacings of under 5 cm to avoid time-delay interference effects. The best results would thus probably be obtained by using four microphone capsules placed in close proximity, and a tetrahedral arrangement of four hypercardioid capsules placed back-to-back should prove satisfactory. In any case, separate bulky microphones would prevent the desired small spacing from being achieved. However, if only reproduction over the tetrahedral loudspeakers of **fig. 5** is required, then the tetrahedral microphones need not be so precisely coincident.

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It is possible to convey and reproduce spaced microphone periphonic recordings via the four channels A^- , B^+ , C^- , and D^+ by pan-potting the outputs of the spaced microphones, so that these outputs appear to come from the desired directions. An audio signal X can be pan-potted to appear to come from any desired direction in space by feeding into each of the four channels the signal that would be picked up by four imaginary coincident tetrahedral hypercardioid microphones were one to imagine the sound of X to be reproduced from a loudspeaker in that direction. One can choose the imaginary tetrahedral microphones' directional characteristic to give the best results in any particular case.

It is particularly easy to pan-pot sounds which are required to appear to come from straight ahead, straight behind, straight above, straight below, or from the left or right side. For instance, a sound X can be made to appear to come from straight ahead by putting $A^- = D^+ = 0$ and $B^+ = C^- = 0.707 X$. (This particular pan-potting simulates the sound that would be picked up by tetrahedral coincident hypercardioids with 125.3° nulls if the sound source were straight ahead). Similarly, a sound X can be made to appear to come from above by putting $A^- = C^- = 0$ and $B^+ = D^+ = 0.707 X$. This simple pan-potting is particularly useful if the recording is made with six microphones at the vertices of an octahedron, pointing forward, backward, to each side, above and below. The sounds from the six microphones can then be pan-potted into position to give a four channel periphonic recording.

It is less simple to pan-pot sounds to appear to come from other directions. If one wishes to make a sound X appear to come from some chosen horizontal direction, the four channels A^- , B^+ , C^- , and D^+ must be the four signals that would be picked up by four imaginary identical coincident cardioids or hypercardioids pointing along *horizontal* directions at 90° to one another, as in fig. 2, if the sound X were to be reproduced through a loudspeaker in the desired direction. Thus, for sounds to be pan-potted in the horizontal plane, one can ignore all three-dimensional considerations. As an example, a sound X may be made to appear to come from 45° to the left by putting $A^- = 0.408 X$, $B^+ = 0.816 X$, $C^- = 0.408 X$, and $D^+ = 0$ (which simulates the sound pick-up of four coincident horizontal cardioid microphones). The tetrahedral symmetry of the channels may be used to derive similar pan-pottings for sounds in the vertical plane pointing forward and backward, or for sounds in the vertical plane pointing sideways. For example, a sound X will appear to come from 45° above straight behind if $A^- = 0.408 X$, $B^+ = 0.408 X$, $C^- = 0$, and $D^+ = 0.816 X$.

It is not all that difficult to pan-pot sounds to come from slightly above or below horizontal. The procedure is first to pan-pot the sound X in the desired direction on the horizontal plane, obtaining four signals A , B , C , and D . One then derives the signals $A^- = A - kX$, $B^+ = B + kX$, $C^- = C - kX$, and $D^+ = D + kX$, where k is a small

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number which is chosen to be positive if the sound is to come from above horizontal, and negative if from below horizontal.

The pan-potting required for a sound to appear to come from the corners of the cube of figs. 5 and 7 may be illustrated by typical examples. A sound X may be made to seem to come from the corner B^+ by putting $B^+ = 0.935 X$ and $A^- = C^- = D^+ = 0.205 X$, which simulates the sound pick-up of 135° -null hypercardioids for a sound source along the B^+ axis. (The signals $B^+ = X$, $A^- = C^- = D^+ = 0$ are not really suitable, as they simulate the sound pick-up of 109.5° -null hypercardioids, and will not reproduce well over a cube of loudspeakers.) A sound X may be made to appear to come from the corner B^- of the cube of figs. 5 and 7 by putting $A^- = B^+ = C^- = 0.570 X$ and $D^+ = -0.160 X$, again simulating the pick-up of 135° -null hypercardioids.

By using means of pan-potting such as described above, the sounds from any number of spaced microphones may be fed into the four periphonic channels A^- , B^+ , C^- , and D^+ .

Conclusions

In the two parts of this article, it has been shown that three channels are sufficient to convey horizontal quadraphonic sound, and four channels sufficient to convey periphonic sound in three dimensions. It has also been shown that it is possible to convey periphonic sound via channels A^- , B^+ , C^- , and D^+ that can be reproduced via the horizontal quadraphonic 'box' speaker layout as in current American proposals, or via a tetrahedral loudspeaker layout giving three-dimensional sound reproduction over an exceptionally large listening area.

In the light of this, it would be wise for recording organisations to include height information on current quadraphonic master-tapes, to allow for the possibility that periphonic systems may become commercial. It would be feasible for companies to start issuing commercial $\frac{1}{4}$ -track quadraphonic tapes conveying periphonic information almost immediately, due to the compatibility of the system described above. In order to ensure standardisation, it is recommended that the front left channel represents the output of an upward inclined microphone, rather than a downward inclined microphone.

It is further recommended that any three or four-channel system adopted for disc, radio or tape should not permit any ambiguity in the polarity of some of the channels with respect to the others, so that it will be possible to matrix signals for the various different loudspeaker layouts.

The author would like to emphasise that the above work is mainly the result of a theoretical analysis. Much remains to be done determining how well the various proposals work with different microphone techniques and different loudspeaker types.

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References

³ Granville Cooper: *Studio Sound*, June 1970.

⁴ Alec Nisbett: Happy Birthday Ludwig! *Studio Sound*, March 1970.

⁵ N. V. Franssen: *Stereophony*, Philips Technical Library, 1964 (pages 25-27).